AP Calculus AB – Worksheet 82 **Optimization and Graphs of f'**

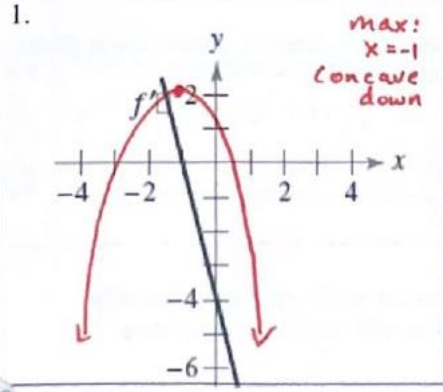
Given $f(x)$ is a differentiable function.

Fill in the blanks to complete the table below

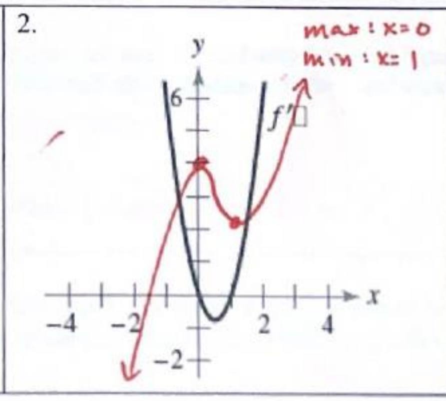
$f(x)$	$f'(x)$	$f''(x)$
$f(x)$ is increasing	$f'(x) > 0$	—
f(x) is decreasing	$f'(x)<0$	—
f(x) has a local max when x=a	$f'(x)$ changes signs from positive to negative when $x=a$.	$f''(a) < 0$
$f(x)$ has a local minimum when $x=a$.	f'(x) changes signs from negative to positive when x = a	$f''(a) > 0$
f(x) is concave down	f'(x) is decreasing	$f''(x)<0$
$f(x)$ is concave up	f'(x) is increasing	$f''(x) > 0$
f(x) has a point of inflection	f'(x) changes from inc to dec or from dec to inc	$f''(x)$ changes signs

The graph of $f'(x)$ is given below. Sketch a possible graph of $f(x)$.

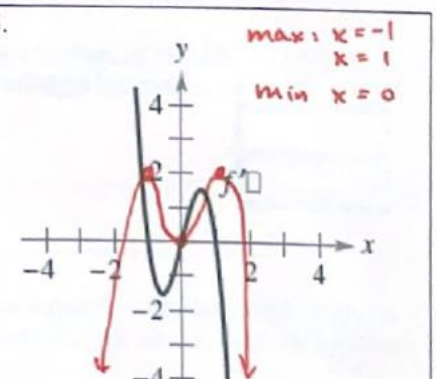
1.



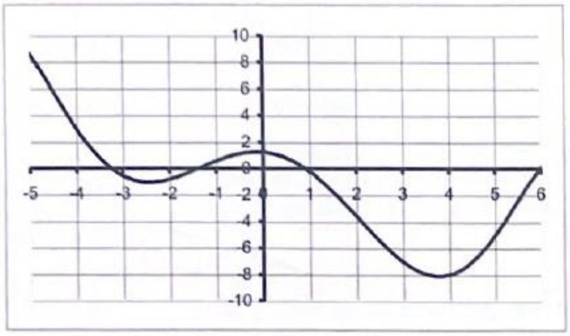
2.



3.



4. A graph of $f'(x)$, the derivative of $f(x)$, is given below. The domain for $f(x)$ is all real numbers.



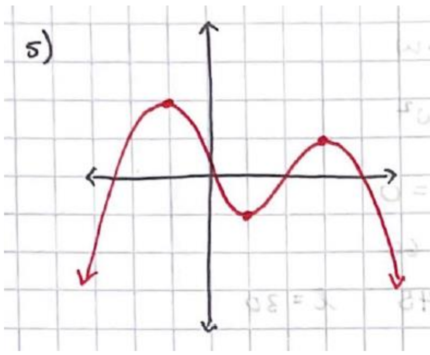
a) On what interval(s) is $f(x)$ increasing? Explain.

$f(x)$ is inc on $(-\infty, -3.2)$ & $(-1.3, 1)$ b/c $f'(x) > 0$.

b) Determine the x-value of any local maximum on the graph of $f(x)$ will occur. Justify your reasoning.

$f(x)$ has a local max at $x = -3.2$ & $x = 1$ b/c $f'(x)$ changes signs from + to −.

5)



6) a) $f(x)$ has a local max @ $x = -2$ b/c $f'(x)$ changes signs from + to −

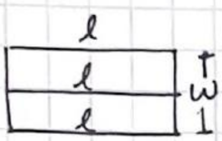
$f(x)$ has a local min @ $x = 0$ b/c $f'(x)$ changes signs from − to +

$f(x)$ has neither max nor min @ $x = 2$ b/c $f'(x)$ does not change signs

b) $f(x)$ is increasing on $[-3, -2)$, $(0, 2)$, & $(2, 3]$ b/c $f'(x) > 0$

c) $f(x)$ is concave down on $(-3, -1)$ & $(1, 2)$ b/c $f''(x) < 0$ ($f'(x)$ is decreasing)

7)



$3l + 2w = 180$

$3l = 180 - 2w$

$l = 60 - \frac{2}{3}w$

$A = l \cdot w$

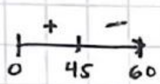
$A = (60 - \frac{2}{3}w) \cdot w$

$A = 60w - \frac{2}{3}w^2$

$A' = 60 - \frac{4}{3}w = 0$

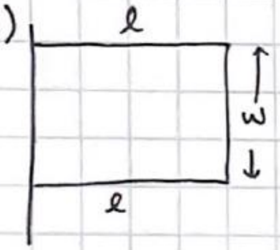
$\frac{4}{3}w = 60$

$w = 45$ $\quad l = 30$



The maximum area that the farmer can enclose is $1350\ ft^2$.

8)



$2l + w = 160$

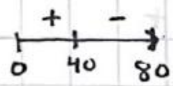
$w = 160 - 2l$

$A = l \cdot w$

$A = l(160 - 2l)$

$A = 160l - 2l^2$

$A' = 160 - 4l = 0$



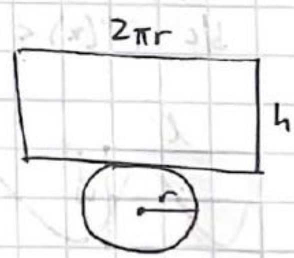
$l = 40$

$w = 80$

The dimensions that will yield the largest area are 40 ft × 80 ft.

9)

Minimize surface Area



$V = \pi r^2 h = 30$

$h = \dfrac{30}{\pi r^2}$

$S = \pi r^2 + 2\pi r h$

$S = \pi r^2 + 2\pi r\left(\dfrac{30}{\pi r^2}\right)$

$S = \pi r^2 + 60 r^{-1}$

$S' = 2\pi r - 60 r^{-2} = 0$

$2\pi r = \dfrac{60}{r^2}$

$r^3 = \dfrac{30}{2\pi}$

$r = \sqrt[3]{\dfrac{15}{\pi}} \quad h = \dfrac{30}{\pi\left(\frac{15}{\pi}\right)^{2/3}}$ cm